ASSESSING AND CONTROLLING THE AVAILABILITY OF FAILURE-DEGRADED SERVICE AGENTS

Institute for Joint Warfare Analysis

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EXECUTIVE SUMMARY

Military items such as airborne surveillance systems (UAVs, JSTARS, helicopters, etc.) or combat vehicles (tanks, APCs, ships) may have high effectiveness when available on station, but require occasional restoration (refueling, re-arming, scheduled maintenance) and repair after unscheduled failures of certain subsystems. This requirement takes them off station, where delays occur that are affected by the numbers and types of support resources and the philosophy of scheduling those resources.

This paper considers the effect of decision choices on long-run item availability on station when items can be in several levels of capability/effectiveness when on station. The model is used to show that a simple binary decision rule (that depends on ratios of endurance, failure, and restoration and repair rates) guides the decision as to whether a failed item should be completely repaired to its highest level, or returned to duty at an incompletely-capable state.

View this as an indicator of the types of rules anticipated to apply in realistic generality. These will be the subject of additional research.

Assessing and Controlling the Availability of Failure-Degraded Service Agents

by

D. P. Gaver and P. A. Jacobs

1. Introduction and Problem Formulation

In a variety of situations several individual items (agents) coexist and perform operational functions, such as region surveillance or actual combat. They require occasional support (e.g. if vehicles, or military "platforms", refueling and repair). Time out for restoration (refueling) is inevitable and unavoidable, but failure of a unit's functional capability or operational condition, while necessary, need not be complete: a partially-failed or degraded unit can profitably remain active under some circumstances. Under certain circumstances, to be described, it is overall-effective to partially repair a completely failed item, rather than expend time and resources for repair to complete functionality. A unique military situation is that in which an item has been subjected to chemical or biological attack, and must be decontaminated: there can be a choice between *partial*, or *complete decontamination*; see Klutke and Novikov (1996). In this paper we provide initial models and rules for the control of all such systems.

Examples of the options and decision choices occur extensively in the military: depending on need for information, it may well be more effective to quickly field partially effective sensor (e.g. UAV) systems than to wait for complete repair. The same can be true of tactically required artillery pieces, tanks, or reconnaissance or attack aircraft; the choice of partial or total decontamination is another example, mentioned above. Other examples occur when prompt assistance is needed after an emergency,

either natural (e.g. earthquake or fire) or terrorist-caused. Failures may now be injuries to humans or infrastructure; partial "repair" means initial diagnosis and stabilization of an injured individual, or provision of temporary alternative communication or transportation service.

The situation modeled is analogous to the classical repairman problem (see Feller (1968)) but is more general – and realistic – in that, for us, the classical "machines", our *agents*, can fail partially, i.e. to one of several intermediate levels of productivity and then, having finally reached the restoration-repair service facility, can be repaired there to chosen levels. This control feature is, in principle, accomplished probabilistically in our models. The control objective is to maximize the expected utility of the system of agents, using as control options the rate of demand for repair, and the choice of repair objective. The utility function involved is a linear function of the numbers of agents available in each performance category. For UAVs this can be expected time on station, weighted for sensor capability.

Our analysis begins by proposing a quasi-stochastic "expected value" or deterministic model for a system of agents that can each be in one of three states of performance capability: *Good*, *Medium*, and *Bad*. When restoration or repair is required the agent progresses to a central homebase server at which accumulated and queued restoration and repair tasks are processed in a weighted processor-sharing manner (this is unrealistic in detail, but is analytically tractable; it will be useful for guiding more detailed simulations). Control is accomplished by choice of the weighting probabilities. This model accounts for restoration-repair facility saturability by using an approximation described in Filipiak (1988). The deterministic model is especially convenient for exploring time-dependent or transient system behavior.

Next, a Markov stochastic network version of the above model is constructed using the techniques of Kelly (1979). The implications of the two model types are compared numerically and seen to be in useful agreement for the utility function adopted.

Our work is related to policies for minimal repair; cf. Brown and Proschan (1983), Phelps (1983), Chen and Feldman (1997), Kijima, Morimura, and Suzuki (1988) and Righter (1997). Also relevant are analytical models of sortie generation, Hackman and Dietz (1997) that address more complex setups in a steady-state context.

2. A Pseudo-Stochastic Deterministic Expected-Value Model

The system will be described in terms of the following state variables. For the present model these are real-valued and positive functions of time; in the subsequent stochastic model they are state variables of a multivariate Markov process in continuous time.

Let

$U_G(t)$ = number of agents in Good condition and Up (productively available) at time t;

$U_M(t)$ = number of agents having experienced a degrading failure and Up at time t; such agents are said to be in Medium condition;

$D_{GB}(t)$ = number of agents that have experienced a mission-affecting failure (MAF) while in Good condition and that are Down for *repair* at time t; such an agent is totally unproductive and is said to be in Bad condition; it will be repaired to the Good state;

$D_{MB}(t)$ = number of agents that are Down at time t because of a mission affecting failure from the Up Medium state;
a decision opportunity: a fraction α_{MG} are sent for further repair to Good, $1 - \alpha_{MG}$ are released to Up Medium state;

$D_{GR}(t)$ = number of agents in Good condition that are Down for restoration (refueling etc.); they will be restored to the Good state;

$D_{MR}(t)$ = number of agents in Medium condition that are Down for restoration to the Medium state at time t;
a decision opportunity: a fraction α_{MR} are sent for further repair to Good, $1 - \alpha_{MR}$ are released to Up Medium state;

$D_{MG}(t)$ = number of agents Down in Medium condition after minimal repair, waiting for repair to the Good state at time t.

Parameters:

N = Total number of agents (e.g. mobile sensors)

λ_{GM} = rate of occurrence of a degrading failure for a Good Up system

λ_{GB} = rate of occurrence of a total failure for an agent (e.g. a sensor) in Good condition

λ_{MB} = rate of occurrence of a total failure for an agent in Medium Up condition

$1/\varepsilon_G$ = mean endurance time for an agent in Good condition

$1/\varepsilon_M$ = mean endurance time for an agent in Medium Up condition

$1/\mu_r(G)$ = mean restoration time for an agent in Good Down condition

$1/\mu_r(M)$ = minimal mean restoration time of an agent in Medium Down condition; agent will remain in Medium condition

$1/\mu_m(B)$ = minimal mean time to repair an agent that has experienced a fatal failure; an agent in Medium condition will still be in Medium condition after this failure

$1/\mu_m(M)$ = mean time to repair an agent for a degrading failure

α_{RG} = fraction of service effort applied to repair a restored Medium Down agent to Good

α_{MG} = fraction of service effort applied to repair a Medium Down agent that had a fatal failure while it was in Good condition

Although the above are *constants* in the developments that follow, they can be replaced by nearly arbitrary *functions of time*, or even state, especially in the quasi-stochastic deterministic model. The differential equations arising in that formulation can be numerically solved, e.g. using MATLAB. This paper is devoted to the steady-state situation, for which analytical solutions are available, and which permit derivation of (almost) optimal control strategies. But transient and also time-dependent solutions are often important and can be obtained more easily with the present approach than using other analytical tools of which we are aware.

Let $S(t)=D_{GR}(t)+D_{MR}(t)+D_{GB}(t)+D_{MB}(t)+D_{MG}(t)$, the number of agents down at time t.

Let $H(x)$ be a differentiable non-increasing function of x; it will represent the amount of service effort when there are x agents waiting for or being served. See Filipiak (1988). An example is

$$H(x) = \frac{1}{1 + c_s x} \qquad (2.1)$$

where c_s is a suitable constant (unity in what follows).

Time-Dependent Equations

$$\frac{dU_G(t)}{dt} = \underbrace{-(\lambda_{GM} + \lambda_{GB} + \varepsilon_G)U_G(t)}_{\text{Failure or Restoration demand from Good state}}$$

$$+ \underbrace{H(S(t))\left[\mu_r(G)D_{GR}(t) + \mu_m(M)D_{MG}(t) + \mu_m(B)D_{GB}(t)\right]}_{\text{Service to Good state}} \qquad (2.2a)$$

$$\frac{dU_M(t)}{dt} = \underbrace{\lambda_{GM}U_G(t)}_{\substack{\text{Partial Failure,}\\\text{Good to Medium}}} - \underbrace{(\varepsilon_M + \lambda_{MB})U_M(t)}_{\substack{\text{Failure to Bad or}\\\text{Restoration demand from Medium}}}$$

$$+ \underbrace{H(S(t))\left[\mu_r(M)(1 - \alpha_{RG})D_{MR}(t) + \mu_m(B)(1 - \alpha_{MG})D_{MB}(t)\right]}_{\text{Restoration or Repair to Medium}} \qquad (2.2b)$$

$$\frac{dD_{GR}(t)}{dt} = \underbrace{\varepsilon_G U_G(t)}_{\substack{\text{Good Restoration}\\\text{demand}}} - \underbrace{H(S(t))\mu_r(G)D_{GR}(t)}_{\substack{\text{Restoration}\\\text{completion to Good}}} \qquad (2.2c)$$

$$\frac{dD_{MR}(t)}{dt} = \underbrace{\varepsilon_M U_M(t)}_{\substack{\text{Medium}\\\text{Restoration demand}}} - \underbrace{H(S(t))\mu_r(M)D_{MR}(t)}_{\substack{\text{Restoration completion}\\\text{to Medium}}} \qquad (2.2d)$$

$$\frac{dD_{MB}(t)}{dt} = \underbrace{\lambda_{MB}U_M(t)}_{\substack{\text{Medium fails}\\\text{to Bad}}} - \underbrace{H(S(t))\mu_m(B)D_{MB}(t)}_{\substack{\text{Repair of Medium, failed to Bad,}\\\text{to Medium}}} \qquad (2.2e)$$

$$\frac{dD_{MG}(t)}{dt} = \underbrace{H(S(t))\left[\mu_r(M)\alpha_{RG}D_{MR}(t) + \mu_m(B)\alpha_{MG}D_{MB}(t)\right]}_{\substack{\text{Number continuing from beyond minimum repair:}\\\text{Medium-repaired to Good repair state}}} - \underbrace{H(S(t))\mu_m(M)D_{MG}(t)}_{\text{Repair to Good completion}} \qquad (2.2f)$$

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$$\frac{dD_{GB}(t)}{dt} = \underbrace{\lambda_{GB}U_G(t)}_{\substack{\text{Total failure,}\\\text{Good to Bad}}} - \underbrace{H(S(t))\mu_m(B)D_{GB}(t)}_{\text{Repair Down Good to Good}} \qquad (2.2\text{g})$$

In Appendix 1 it is shown that the solution to the steady-state equations obtained from (2.2) by setting derivatives equal to zero is

$$U_M = U \cdot 1/(1+\kappa) \qquad (2.3)$$

$$U_G = U \cdot \kappa/(1+\kappa) \qquad (2.4)$$

where U is the modified solution of a quadratic:

$$U = \frac{2N}{\left[1+(1+c_sN)c/(1+\kappa)\right]+\sqrt{\left[1+(1+c_sN)c/(1+\kappa)\right]^2 - 4c_sNc/(1+\kappa)}} \qquad (2.5)$$

$$\kappa = (\alpha_{RG}\varepsilon_M + \alpha_{MG}\lambda_{MB})/\lambda_{GM} \qquad (2.6)$$

$$c = \frac{\varepsilon_M}{\mu_r(M)} + \frac{\lambda_{MB}}{\mu_m(B)} + \kappa\left(\frac{\varepsilon_G}{\mu_r(G)} + \frac{\lambda_{GM}}{\mu_m(M)} + \frac{\lambda_{GB}}{\mu_m(B)}\right) \qquad (2.7)$$

2.1 Repair Control for the Pseudo-Stochastic Model

Suppose the expected utility of the system of agents is written as

$$g(\alpha, \beta, N) = U_G(\alpha, N) + \beta\, U_M(\alpha, N)$$

where we let $\alpha_{RG} = \alpha_{MG} = \alpha$, the probability that an agent that has been Down and has been restored/repaired to Medium level is allocated further repair to the Good state, with $0 \le \beta < 1$. This notation makes explicit the degradation of capability associated with being in the Medium rather than Good state, and also the control inherent in the probability α; *finally*, the influence of the agent population size, N, is expressed.

Given values for β and N in addition to other rate parameters, the utility function g can be numerically studied/optimized on α. Considerable numerical exploration, using random selections of system parameters in a manner analogous to that suggested by Miller (1997), leads to the empirical conclusion that random strategies $(0 < \alpha < 1)$ are

rarely optimal, and, when they do occur are only slightly better than pure strategies ($\alpha = 0$, or $\alpha = 1$). In fact, consideration of the case N large clarifies the situation and leads to the nearly-optimal control policy as follows:

Let

$$a = \frac{\varepsilon_M}{\mu_r(M)} + \frac{\lambda_{MB}}{\mu_m(B)} \tag{2.8}$$

$$b = \frac{\varepsilon_G}{\mu_r(G)} + \frac{\lambda_{GM}}{\mu_m(M)} + \frac{\lambda_{GB}}{\mu_m(B)} \tag{2.9}$$

$$\gamma = \frac{\varepsilon_M + \lambda_{MB}}{\lambda_{GM}} \tag{2.10}$$

Then $\kappa = \gamma\alpha$ and $c = a + b\gamma\alpha$, the parameters in the solution U.

When N becomes large,

Optimum $\alpha = 1$ if $\beta < a/b$;

Optimum $\alpha = 0$ if $\beta > a/b$. $\tag{2.11}$

To show this, let $N \rightarrow \infty$ in expression (2.5) for U to obtain

$$U(\infty) = \frac{1+\kappa}{c_s c} = \frac{1+\gamma\alpha}{c_s(a+b\gamma\alpha)}$$

so, by (2.3) – (2.4)

$$g(\alpha; \beta, \infty) = \left(\frac{1+\kappa}{c_s c}\right)\left[\frac{\kappa}{(1+\kappa)} + \frac{\beta}{(1+\kappa)}\right]$$

$$= \frac{\gamma\alpha + \beta}{c_s(b\gamma\alpha + a)}.$$

Since

$$\frac{dg}{d\alpha} = \frac{\gamma(a - b\beta)}{(b\gamma\alpha + a)^2}$$

the above policy is verified.

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3. A Stochastic Model

Consider a closed queuing network with N agents (e.g. mobile sensors). There are two service classes: up (U) and down (D). An agent chooses a type each time it leaves the down service class in good condition. The agent types are

G: is in good condition

M: is in medium condition

The expected time an agent of type i spends in the U class is

$$\frac{1}{\upsilon_G} = \frac{1}{\varepsilon_G + \lambda_{GM} + \lambda_{GB}}.$$

(3.1)

Let $a_G(U)$ be the expected amount of time an agent spends up in good condition before it is reclassified; let $a_M(U)$ be the conditional expected amount of time an agent spends up in medium condition before it is reclassified, given it spends time in Medium condition; let $a(U)$ be the total expected time an agent spends in the up state before it is reclassified. For $0 < \alpha_{RG} + \alpha_{MG}$

$$a_M(U) = \frac{1}{\varepsilon_M + \lambda_{MB}} \frac{1}{\dfrac{\varepsilon_M}{\varepsilon_M + \lambda_{MB}} \alpha_{RG} + \dfrac{\lambda_{MB}}{\varepsilon_M + \lambda_{MB}} \alpha_{MG}}$$

(3.2)

$$a_G(U) = \frac{1}{\varepsilon_G + \lambda_{GB} + \lambda_{GM}}$$

(3.3)

$$a(U) = a_G(U) + \frac{\lambda_{GM}}{\varepsilon_G + \lambda_{GB} + \lambda_{GM}} a_M(U).$$

(3.4)

Let $a_G(D)$ be the expected amount of time an agent spends down with a MAF but is in otherwise good condition before it is reclassified; let $a_M(D)$ be the conditional expected amount of time an agent spends down in Medium condition before it is reclassified, given it spends time in Medium condition; let $a_{MR}(D)$ be the conditional expected amount of time a Medium condition an agent spends in restoration before being reclassified, given it spends time in Medium condition; let $a_{MM}(D)$ be the conditional expected amount of time an agent in Medium condition spends being repaired for MAF before being reclassified,

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given it spends time in Medium condition; let $a(D)$ be the expected amount of time an agent spends in Down state before it is reclassified. For $0 < \alpha_{RG} + \alpha_{MG}$

$$a_M(D) = \frac{\dfrac{\varepsilon_M}{\varepsilon_M + \lambda_{MB}} \dfrac{1}{\mu_r(M)} + \dfrac{\lambda_{MB}}{\varepsilon_M + \lambda_{MB}} \dfrac{1}{\mu_m(B)}}{\dfrac{\varepsilon_M}{\varepsilon_M + \lambda_{MB}} \alpha_{RG} + \dfrac{\lambda_{MB}}{\varepsilon_M + \lambda_{MB}} \alpha_{MG}} + \frac{1}{\mu_m(M)} \tag{3.5}$$

$$a_G(D) = \frac{\varepsilon_G}{\varepsilon_G + \lambda_{GB} + \lambda_{GM}} \frac{1}{\mu_r(G)} + \frac{\lambda_{GB}}{\varepsilon_G + \lambda_{GB} + \lambda_{GM}} \frac{1}{\mu_m(G)} \tag{3.6}$$

$$a(D) = a_G(D) + \frac{\lambda_{GM}}{\varepsilon_G + \lambda_{GB} + \lambda_{GM}} a_M(D) \tag{3.7}$$

$$a_{MR}(D) = \frac{\dfrac{\varepsilon_M}{\varepsilon_M + \lambda_{MB}} \dfrac{1}{\mu_r(M)}}{\dfrac{\varepsilon_M}{\varepsilon_M + \lambda_{MB}} \alpha_{RG} + \dfrac{\lambda_{MB}}{\varepsilon_M + \lambda_{MB}} \alpha_{MG}} \tag{3.8}$$

$$a_{MB}(D) = \frac{\dfrac{\lambda_{MB}}{\varepsilon_M + \lambda_{MB}} \dfrac{1}{\mu_m(M)}}{\dfrac{\varepsilon_M}{\varepsilon_M + \lambda_{MB}} \alpha_{RG} + \dfrac{\lambda_{MB}}{\varepsilon_M + \lambda_{MB}} \alpha_{MG}} \tag{3.9}$$

Theorem (3.12) of Kelly (1979) implies that the limiting distribution of the number of agents in the up class and the down class is

$$\pi_{U,D}(n_U, n_D) = K \frac{a(U)^{n_U}}{n_U!} \frac{a(D)^{n_D}}{\displaystyle\prod_{k=1}^{n_D} \min(k,s)} \tag{3.10}$$

where K is a normalizing constant and s is the number of servers in the D class. The expected number of agents in the up class (respectively down class) is

$$E[U] = K_U \sum_{n_U=0}^{N} n_U \frac{a(U)^{n_U}}{n_U!} \tag{3.11}$$

$$\text{(respectively } E[D] = K_D \sum_{n_D=0}^{N} n_D \frac{a(D)^{n_D}}{\displaystyle\prod_{k=1}^{n_D} \min(k,s)}) \tag{3.12}$$

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where K_U and K_D are normalizing constants. The expected number of agents in the up class that are in good condition (respectively medium condition) is

$$E[U_G] = E[U]\frac{a_G(U)}{a(U)}$$ (3.13)

(respectively, $E[U_M] = E[U]\frac{a_M(U)}{a(U)}$) (3.14)

The expected number of agents in the down class that are being repaired for an MAF but are otherwise in good condition (respectively are in medium condition)

$$E[D_G] = E[D]\frac{a_G(D)}{a(D)}$$ (3.15)

(respectively, $E[D_M] = E[D]\frac{a_M(D)}{a(D)}$) (3.16)

Numerical Examples

Below are tables comparing the numbers of agents in various categories for the deterministic and stochastic models. While the two models are not exactly comparable, the deterministic model yields results close to those of the stochastic model.

Table 1

N=4; λ_{GM}=0.1; λ_{GB}=0.05; λ_{MB}=0.05; ε_G=0.2; ε_M=0.2; $\mu_m(M)$=0.5; $\mu_m(G)$=0.5; $\mu_m(MG)$=5; $\mu_r(G)$=10; $\mu_r(M)$=10; α_{RG}=0.5; α_{MG}=0.5; single server; c_s=1

Expected Number

Category	Deterministic	Queuing
	Model	Model
Good: Up	1.81	1.87
Medium: Up	1.45	1.50
Good: Catastrophic Failure	0.32	0.27
Medium: Catastrophic Failure	0.25	0.21
Repair: Medium to Good	0.06	0.05
Good: Restore	0.06	0.05
Medium: Restore	0.05	0.04

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Table 2

$N=4$; $\lambda_{GM}=1$; $\lambda_{GB}=0.05$; $\lambda_{MB}=0.1$; $\varepsilon_G=0.2$; $\varepsilon_M=0.5$; $\mu_m(M)=0.5$; $\mu_m(G)=0.5$; $\mu_m(MG)=5$;
$\mu_r(G)=10$; $\mu_r(M)=7$; $\alpha_{RG}=0.5$; $\alpha_{MG}=0.5$; single server; $c_s=1$

Expected Number

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Category	Deterministic Model	Queuing Model
Good: Up	0.52	0.60
Medium: Up	1.73	2.00
Good: Catastrophic Failure	0.14	0.11
Medium: Catastrophic Failure	0.95	0.76
Repair: Medium to Good	0.29	0.23
Good: Restore	0.03	0.02
Medium: Restore	0.34	0.27

4. The Repair Policy for the Stochastic Model with $N=1$

Let

$$E[U(\alpha)] = \frac{1}{\lambda_{GB}+\varepsilon_G+\lambda_{GM}} + \frac{\lambda_{GM}}{\lambda_{GB}+\varepsilon_G+\lambda_{GM}}\frac{1}{\alpha}\frac{1}{\lambda_{MB}+\varepsilon_M} \tag{4.1}$$

$$E[D(\alpha)] = \frac{\lambda_G}{\lambda_{GB}+\varepsilon_G+\lambda_{GM}}\frac{1}{\mu_m(B)} + \frac{\varepsilon_G}{\lambda_{GB}+\varepsilon_G+\lambda_{GM}}\frac{1}{\mu_r(G)}$$

$$+ \frac{\lambda_{GM}}{\lambda_{GB}+\varepsilon_G+\lambda_{GM}}\left[\frac{1}{\mu_m(M)} + \frac{1}{\alpha}\left[\frac{\varepsilon_M}{\lambda_{MB}+\varepsilon_M}\frac{1}{\mu_r(M)} + \frac{\lambda_{MB}}{\lambda_{MB}+\varepsilon_M}\frac{1}{\mu_m(B)}\right]\right] \tag{4.2}$$

$$g(\alpha;\beta) = \frac{\dfrac{1}{\lambda_{GB}+\varepsilon_G+\lambda_{GM}} + \beta\dfrac{\lambda_{GM}}{\lambda_{GB}+\varepsilon_G+\lambda_{GM}}\dfrac{1}{\alpha}\dfrac{1}{\lambda_{MB}+\varepsilon_M}}{E[U(\alpha)+D(\alpha)]}$$

$$= \frac{\alpha + \beta\dfrac{\lambda_{GM}}{\lambda_{GM}+\varepsilon_M}}{\alpha\left[1+\dfrac{\lambda_{GB}}{\mu_m(B)}+\dfrac{\varepsilon_G}{\mu_r(G)}+\dfrac{\lambda_{GM}}{\mu_m(M)}\right] + \dfrac{\lambda_{GM}}{\lambda_{MB}+\varepsilon_M}\left[\dfrac{\varepsilon_M}{\mu_r(M)}+\dfrac{\lambda_M}{\mu_m(B)}\right]} \tag{4.3}$$

$$= \frac{\alpha + \beta\dfrac{1}{\gamma}}{\alpha[1+b]+\dfrac{1}{\gamma}a}$$

Note that

$$\frac{dg(\alpha;\beta)}{d\alpha} = \frac{(1+b)\left[\dfrac{a}{1+b}-\beta\right]\dfrac{1}{\gamma}}{\left(\alpha[1+b]+\dfrac{1}{\gamma}a\right)^2}$$ (4.4)

Hence the repair policy is as follows.

The Repair Policy for Stochastic Model with *N*=1:

If $\beta < \dfrac{a}{1+b}$ then the maximizing α is $\alpha=1$; always repair the agent in Medium condition to Good.

If $\beta > \dfrac{a}{1+b}$ then the maximizing α is $\alpha=0$; never repair the agent in Medium condition to Good.

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APPENDIX 1
Long-Run Solutions

Setting the derivatives equal to zero in system (2a) – (2g) results in the following equations

$$0 = -(\lambda_{GM} + \lambda_{GB} + \varepsilon_G)U_G$$
$$+ H(S)\big[\mu_r(G)D_{GR} + \mu_m(M)D_{MG} + \mu_m(B)D_{GB}\big] \tag{A1-1a}$$

$$0 = \lambda_{GM}U_G - (\varepsilon_M + \lambda_{MB})U_M$$
$$+ H(S)\big[\mu_r(M)(1 - \alpha_{RG})D_{MR} + \mu_m(B)(1 - \alpha_{MG})D_{MB}\big] \tag{A1-1b}$$

$$0 = \varepsilon_G U_G - H(S)\mu_r(G)D_{GR} \tag{A1-1c}$$

$$0 = \varepsilon_M U_M - H(S)\mu_r(M)D_{MR} \tag{A1-1d}$$

$$0 = \lambda_{MB}U_M - H(S)\mu_m(B)D_{MB} \tag{A1-1e}$$

$$0 = H(S)\big[\mu_r(M)\alpha_{RG}D_{MR} + \mu_m(B)\alpha_{MG}D_{MB}\big] - H(S)\mu_m(M)D_{MG} \tag{A1-1f}$$

$$0 = \lambda_{GB}U_G - H(S)\mu_m(B)D_{GB} \tag{A1-1g}$$

These must be solved subject to the condition that all state values sum to N. Equations (A1-1d), (A1-1e), and (A1-1g) result in

$$D_{MB} = \frac{\lambda_{MB}U_M}{H(S)\mu_m(B)} \tag{A1-2}$$

$$D_{MR} = \frac{\varepsilon_M U_M}{H(S)\mu_r(M)} \tag{A1-3}$$

$$D_{GB} = \frac{\lambda_{GB}U_G}{H(S)\mu_m(B)} \tag{A1-4}$$

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Substitution of (A1-3) and (A1-2) into equation (A1-1b) results in

$$0 = \lambda_{GM} U_G - (\varepsilon_M + \lambda_{MB}) U_M$$
$$+ \left[(1 - \alpha_{RG}) \varepsilon_M U_M + (1 - \alpha_{MG}) \lambda_{MB} U_M \right] \qquad \text{(A1-5)}$$

Thus

$$U_G = \kappa U_M \qquad \text{(A1-6)}$$

where

$$\kappa = \frac{\alpha_{RG} \varepsilon_M + \alpha_{MG} \lambda_{MB}}{\lambda_{GM}} \qquad \text{(A1-7)}$$

if $\lambda_{GM} > 0$.

Substitution of (A1-3) and (A1-2) into equation (A1-1f) results in

$$0 = \varepsilon_M \alpha_{RG} U_M + \lambda_{MB} \alpha_{MG} U_M - H(S) \mu_m(M) D_{MG} \qquad \text{(A1-8f)}$$

Multiplying equation (A1-1d) by $\mu_r(G)/\mu_r(M)$, equation (A1-1e) by $\mu_r(G)/\mu_m(B)$, equation (A1-8f) by $\mu_r(G)/\mu_m(M)$ and equation (A1-1g) by $\mu_r(G)/\mu_m(B)$, adding the resulting equations and using (A1-6) results in

$$c U_M = H(S) S \qquad \text{(A1-9)}$$

where

$$c = \frac{\varepsilon_G \kappa}{\mu_r(G)} + \frac{\varepsilon_M}{\mu_r(M)} + \frac{\lambda_{MB}}{\mu_m(B)} + \frac{\lambda_{GM} \kappa}{\mu_m(M)} + \frac{\lambda_{GB} \kappa}{\mu_m(B)} \qquad \text{(A1-10)}$$

Note that

$$S = N - (1 + \kappa) U_M \qquad \text{(A1-11)}$$

Thus (A1-9) can be rewritten as

$$c U_M = \left[N - (1 + \kappa) U_M \right] H \left(N - (1 + \kappa) U_M \right) \qquad \text{(A1-12)}$$

Putting $U = U_G + U_M = (1 + \kappa) U_M$ and substituting into equation (A1-12) results in

$$\frac{c}{1+\kappa}U = [N-U]H(N-U) \qquad \text{(A1-13)}$$

Rewriting,

$$\frac{c}{1+\kappa}\frac{U}{[N-U]} = H(N-U) \qquad \text{(A1-14)}$$

Equation (A1-14) has one solution between 0 and N.

In the special case $H(x)=1/(1+c_sx)$ equation (A1-14) is a quadratic

$$0 = \frac{c_sc}{1+\kappa}U^2 - (1+\frac{c}{1+\kappa}(1+c_sN))U + N \qquad \text{(A1-15)}$$

$$U = \frac{(1+\frac{c}{1+\kappa}(1+c_sN)) - \sqrt{\left[(1+\frac{c}{1+\kappa}(1+c_sN))^2 - 4\frac{c_sc}{1+\kappa}N\right]}}{2\frac{c_sc}{1+\kappa}} \qquad \text{(A1-16)}$$

Finally

$$U_M = U/(1+\kappa)$$

and

$$U_G = \frac{\kappa}{1+\kappa}U.$$

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APPENDIX 2

A Repair Policy for the Deterministic Expected-Value Model

Let $\alpha_{RG}=\alpha_{MG}=\alpha$. Put

$$a = \frac{\varepsilon_M}{\mu_r(M)} + \frac{\lambda_{MB}}{\mu_m(B)} \tag{A2-1}$$

$$b = \frac{\varepsilon_G}{\mu_r(G)} + \frac{\lambda_{GM}}{\mu_m(M)} + \frac{\lambda_{GB}}{\mu_m(B)} \tag{A2-2}$$

$$\gamma = \frac{\varepsilon_M + \lambda_{MB}}{\lambda_{GM}} \tag{A2-3}$$

Note that $\kappa=\gamma\alpha$ and $c=a+b\gamma\alpha$. Thus,

$$\frac{c}{1+\kappa} = \frac{a+b\gamma\alpha}{1+\gamma\alpha} = a + \frac{\gamma(b-a)\alpha}{1+\gamma\alpha}. \tag{A2-4}$$

Let $U(\alpha)$ be the solution to equation (A1-9).

Consider the weighted (expected) number of agents up

$$\begin{aligned} g(\alpha;\beta) &= U_G(\alpha) + \beta U_M(\alpha) \\ &= \frac{\gamma\alpha+\beta}{1+\gamma\alpha} U(\alpha) \end{aligned} \tag{A2-5}$$

for $0 \leq \beta \leq 1$.

Policy I: This maximizes a utility function that assigns a relatively low value to units in the Medium state.

If $b<a$, then $g(\alpha;\beta)$ is maximized at $\alpha=1$; always repair an agent in Medium condition to Good.

If $b>a$, then there is a $0 < \beta_0 < \beta_1 \leq 1$ such that for $\beta \leq \beta_0$ $g(\alpha;\beta)$ is maximized at $\alpha=1$ (always repair a Medium agent to Good); for $\beta \geq \beta_1$ $g(\alpha;\beta)$ is maximized at $\alpha=0$ (never repair a Medium agent to Good).

Policy II: This maximizes the long-run total number of agents up

If $b<a$, then $U(\alpha)$ is maximized at $\alpha=1$; that is always repair an agent in the Medium condition to Good.

If $b>a$, then $U(\alpha)$ is maximized at $\alpha=0$; that is never repair an agent in Medium condition to Good.

Proof for Policy II:

Note that $f(\alpha) = \dfrac{c}{1+\kappa}$ is a decreasing function of α if $b<a$ and is a nondecreasing function of α otherwise. Since $H(x)$ is a decreasing function, $H(N-U)$ is an increasing function of U. Further, $(N-U)H(N-U)$ is a decreasing function of U. If $b<a$ so that $f(\alpha)$ decreases with α, then equation (A1-13) implies that $U(\alpha)$ increases with α. If $b>a$, then $f(\alpha)$ increases with α and $U(\alpha)$ decreases with α. Policy I follows.

Proof for Policy I:

Note that

$$\frac{d}{d\alpha}g(\alpha;\beta) = \frac{\gamma(1-\beta)}{1+\gamma\alpha}U(\alpha) + \frac{\gamma\alpha+\beta}{1+\gamma\alpha}\frac{dU(\alpha)}{d\alpha}$$

$$= \frac{\gamma}{1+\gamma\alpha}U(\alpha) + \frac{\gamma\alpha}{1+\gamma\alpha}\frac{dU(\alpha)}{d\alpha}$$

$$+ \frac{\beta}{1+\gamma\alpha}\left[-\gamma U(\alpha) + \frac{dU(\alpha)}{d\alpha}\right]$$

If $b<a$, then $\dfrac{dU(\alpha)}{d\alpha}>0$ and $\dfrac{d}{d\alpha}g(\alpha;\beta)>0$; thus the maximizing α for $g(\alpha;\beta)$ is $\alpha=1$.

If $b>a$, then $\dfrac{dU(\alpha)}{d\alpha}<0$. If $\beta=0$, then $\gamma(\alpha;0)=U_G(\alpha)$. Recall that

$$U_G(\alpha) = \gamma\alpha U_M(\alpha).$$

Thus,

$$\frac{d}{d\alpha}U_G(\alpha) = \gamma U_M(\alpha) + \gamma\alpha\frac{d}{d\alpha}U_M(\alpha)$$

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and

$$\frac{d}{d\alpha}U(\alpha) = \frac{d}{d\alpha}U_G(\alpha) + \frac{d}{d\alpha}U_M(\alpha)$$

$$= \gamma U_M(\alpha) + [\gamma\alpha + 1]\frac{d}{d\alpha}U_M(\alpha).$$

Hence,

$$\frac{d}{d\alpha}U_M(\alpha) = \frac{\dfrac{d}{d\alpha}U(\alpha) - \gamma U_M(\alpha)}{\gamma\alpha + 1} < 0.$$

The value of α which minimizes $U_M(\alpha)$ is $\alpha=1$. In addition $\dfrac{d}{d\alpha}U_M(\alpha) < \dfrac{d}{d\alpha}U(\alpha)$. Since $\dfrac{d}{d\alpha}U(\alpha) = \dfrac{d}{d\alpha}U_G(\alpha) + \dfrac{d}{d\alpha}U_M(\alpha)$ it follows that $0 < \dfrac{d}{d\alpha}U(\alpha) - \dfrac{d}{d\alpha}U_M(\alpha) = \dfrac{d}{d\alpha}U_G(\alpha)$.

Thus, the maximizing value of α for $U_G(\alpha)$ is $\alpha=1$; always repair agents in Medium condition to Good. Thus,

$$\frac{\gamma}{1+\gamma\alpha}U(\alpha) + \frac{\gamma\alpha}{1+\gamma\alpha}\frac{dU(\alpha)}{d\alpha} > 0.$$

Recall from Policy II that if $\beta=1$ then $\dfrac{d}{d\alpha}g(\alpha;1) < 0$ and the maximizing α is $\alpha=0$ (never repair a Medium agent to Good condition). Policy I follows.

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